Online Appendix for SWFF-DFM and SW-DFM Models

Sacha Gelfer*

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Abstract

I consider two DSGE models in this online appendix, the first model is based on the FRBNY model outlined by Del Negro et al. (2013). This model is an extension of the Smets and Wouters (2003, 2007) New Keynesian model with the addition of a credit market with frictions that closely follows the financial accelerator model created by Bernanke, Gertler and Gilchrist (1999). It incorporates many of the features of Christiano, Motto and Rostagno (2010). The second model has no credit channel and closely follows the Smets and Wouters (2003) model. This model will be referred to as SW while the model with financial frictions will be referred to as SWFF. This online appendix proceeds as follows, I first outline the agents in the SWFF model and discuss their choices and optimization problems. Next, I present the linearized equations of the model around the steady state that I use to produce my results. Finally, I introduce the components of the SW model that differ from the SWFF model, as well as any linearized equations that change as a result of how the SW model is microfounded.

* *Bentley University*, Waltham, MA, USA. E-mail address: sgelfer@bentley.edu

1 The DSGE Models

1.1 General Outline of SWFF Model

The model involves a number of exogenous shocks, economic agents, and market frictions. The agents include households, intermediate and wholesale firms, banks, entrepreneurs, capital producers, employment agencies, and government agencies. The agents and their choice behavior decisions along with what shocks impact which agents directly are illustrated in Figure 1.

Households supply household-specific labor to employment agencies. Households maximize a CRRA utility function over an infinite horizon with additively separable utility in consumption, leisure and money. Utility from consumption has habit persistence as it is realized by a relative measure of total consumption in the last time period. Labor is differentiated over households, and is not perfectly competitive implying households hold some monopoly power over wages. The model includes sticky nominal wages set in a Calvo (1983) manner with wage indexation to those who can not freely optimize their wage. In addition to holding money, households can save in Government bonds and/or deposits in banks. Households are subject to an exogenous preference shock that can be viewed as a shock in the consumer's consumption and saving decisions.

Employment Agencies package and sell labor bought from the household to intermediate-firms. Employment agencies are perfectly competitive but must buy specialized labor from households who hold some monopoly power over wages. Households and Employment Agencies may only renegotiate wages with a certain probability but are subject to inflation indexation. Employment agencies are subject to wage mark-up shocks that capture exogenous changes in the monopolistic power households hold over their specialized labor.

Firms come in two forms, intermediate good producing firms and final good producing firms. There is a continuum of intermediate good firms, who supply intermediate goods in a monopolistically competitive market. Intermediate firms produce differentiated goods, decide on labor and capital inputs, and set prices in a Calvo-like manner. As with wages, those firms unable to change their prices, are able to partially index them to past inflation rates. Intermediate firms face two exogenous shocks, the first is a productivity shock that affects their production ability and the second is a price mark-up shock. The price mark-up shock captures the degree of competitiveness in the intermediate goods market. Final goods use intermediate goods in production and are produced in perfect competition. The final good is sold to the households and capital producers in the form of consumption.

Capital Producers buy consumption output from the final goods sector and transform it into new capital. The creation of new capital (Investment) requires both the newly bought consumption output and the previous stock of capital in the economy which they buy from entrepreneurs. The investment procedure is subject to convex adjustment costs making it more expensive to produce more capital in times of large investment growth. Capital producers are subject to investment shocks that affect the marginal efficiency of investment as in Justiniano et al. (2011).

Financial Sector centers around two economic agents, banks and entrepreneurs. Entrepreneurs enter the period with some level of net worth. They must use their net worth and an agreed upon loan from the bank to buy capital from the capital producers. Once the capital is bought they are affected by an idiosyncratic risk shock that can decrease or increase their overall level of capital just purchased. The entrepreneur must then decide the utilization of the new level of capital and rent it out to intermediate firms to be used in their production process. Once the capital has been used in the production process the non-depreciated capital is purchased by the capital producers. If entrepreneurs received enough revenue they pay back the agreed upon loan with interest to the bank. If entrepreneurs do not have enough revenue a proportion of their revenue is seized by the bank. Banks incorporate the risk of default by charging entrepreneurs an interest rate higher than the deposit rate payed to households. Entrepreneurs face a probability of death after each time period and the banking sector is perfectly competitive. Figure 2 describes the sequence of events amongst all relevant agents when it comes to the financial sector.

Government Agencies are composed of a monetary authority and a fiscal authority. The short term nominal interest rate is determined by the monetary authority, which is assumed to follow a generalized Taylor Rule and is subject to monetary policy shocks. The monetary authority supplies the corresponding money demanded by the household to support the targeted nominal interest rate. The fiscal authority sets government spending and collects lump sum taxes. It is subject to exogenous government spending shocks. Finally, there is a resource constant that states that all final output must equal consumption, investment, government purchases, loan monitoring costs and capital utilization costs.

Let's now examine each economic agents optimization problem and constraint. All relevant first order conditions can be found in the online appendix.

1.1.1 Households

There is a continuum of households indexed by j. The objective function for household j is given by:

$$E_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[\frac{(C_{t+s}(j) - hC_{t+s-1})^{1-\sigma_c}}{1-\sigma_c} - \frac{(L_{t+s}(j))^{1+\nu_l}}{1+\nu_l} + log\left(\frac{M_{t+s}(j)}{P_{t+s}}\right)\right] \tag{1.1}$$

where $C_t(j)$ is household consumption, $L_t(j)$ is supply of a household differentiated type of Labor and $M_t(j)$ is household money holdings. Households face a stochastic shock b_t that can be viewed as an intertemporal preference shock that creates a wedge between the marginal utility of consumption and the real return to risk-free government bonds. h is an identical parameter across households that captures consumption persistence. All parameters not indexed by j are assumed to be identical across all households. Household j's budget constraint is:

$$\begin{aligned} P_{t+s}C_{t+s}(j) + B_{t+s}(j) + D_{t+s}(j) + M_{t+s}(j) \leq R_{t+s-1}B_{t+s-1}(j) \\ + R^d_{t+s-1}D_{t+s-1}(j) + M_{t+s-1}(j) + W_{t+s}(j)L_{t+s}(j) + \Pi_{t+s}(j) - T_{t+s} + Trans_{t+s} \end{aligned} \tag{1.2}$$

where P_t is the price index of the economy, $B_t(j)$ is holdings of government bonds, $D_t(j)$ is the amount of deposits in the banking sector, R_t is the nominal interest rate on government bonds, R^d_t is the nominal interest rate banks pay on deposits, Π_t is the profit households get from owning the intermediate firms, $W_t(j)$ is the wage earned, T_t are lump sum taxes payed to/by the government and $Trans_t$ are wealth transfers to/from the entrepreneurial agents. Household j chooses $\{C_t(j), L_t(j), M_t(j), B_t(j), D_t(j)\}_{t=0}^{\infty}$ that maximizes expected utility (1.1) subject to the household budget constraint (1.2). Further, households may purchase state-contingent securities (not indicated in the budget constraint) which implies that all households choose the same amount of consumption, money holdings, bond purchases and bank deposits.

1.1.2 Employment Agencies

Households sell their specialized labor $L_t(j)$ to employment agencies who then bundle it and sell it to intermediate firms as L_t. The composite labor good of the economy is a CES aggregator of the households specialized labor.

$$L_t = \left(\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} \, dj \right)^{1+\lambda_{w,t}} \tag{1.3}$$

The parameter $\lambda_{w,t}$ is a stochastic process centered around λ_w that measures the monopoly power a household holds in selling its specialized labor. The first order condition of the agencies' profit maximization problem leads to the following demand for specialized labor $L_t(j)$:

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{1.4}$$

Households choose the optimal wage subject to the labor demand function. However, in every time period a probability exists ξ_w, that households can not freely readjust their wage. If a household can not readjust their wage, their wage is automatically indexed to a weighted average of steady state inflation and last periods inflation as in Erceg, Henderson, and Levin (2000).

$$W_t(j) = \left(\pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \right) W_{t-1}(j) \tag{1.5}$$

For households who are able to adjust $W_t(j)$, they face the following optimization problem:

$$\max_{W_t^*(j)} \quad E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \left[-\frac{b_{t+s} L_{t+s}(j)^{1+\nu_L}}{1+\nu_L} + \Lambda_{t+s} W_t(j) L_{t+s}(j) \right] \tag{1.6}$$

$$s.t \qquad \text{equation 1.4 and}$$

$$W_{t+s}(j) = \prod_{k=1}^{s} \left(\pi_{t+k-1}^{\iota_w} \pi^{1-\iota_w} \right) W_t(j) \tag{1.7}$$

Households are maximizing the expected discounted utility from consuming future wage income minus the expected discounted disutility of all future labor while factoring in their labor demand rule and wage indexation rule. (Λ_t is the Lagrange multiplier associated with the households' budget constraint.)

1.1.3 Final Good Producers

Final good producers operate in a perfectly competitive market. They buy intermediate goods $Y_t(i)$, package them into final output Y_t and resell it to consumers. The final good of the economy is a CES production function of a continuum of intermediate goods.

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{f,t}}} \, dj \right)^{1+\lambda_{f,t}} \tag{1.8}$$

The parameter $\lambda_{f,t}$ is a stochastic process centered around λ_f that gauges the monopoly power an intermediate firm has in selling its specific good i. The first order condition of the final good producers profit maximization problem leads to the following demand for good $Y_t(i)$:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} Y_t \tag{1.9}$$

1.1.4 Intermediate Good Producers

Intermediate good producers are the first stage of production. Intermediate firms use utilized capital and labor packaged by the employment agencies to produce differentiated intermediate goods that they sell to the final goods producers. A continuum of these firms indexed by i exist and use the following production process:

$$Y_t(i) = \varepsilon_t^a K_t(i)^\alpha L_t(i)^{1-\alpha} - f \tag{1.10}$$

where f is a fixed cost of the production process, K_t is utilized capital and ε_t^a is a stationary stochastic productivity shock that alters the production process. Firms hire labor and rent capital in perfectly competitive markets and pay identical wages and rental rates. The intermediate firms' profit is given by:

$$P_t(i)\left(\varepsilon_t^a K_t(i)^\alpha L_t(i)^{1-\alpha} - f\right) - W_t L_t(i) - R_t^k K_t(i) \tag{1.11}$$

Intermediate firms choose the optimal price to sell their intermediate good i subject to good's demand function. However, in every time period a probability exists ξ_p that a firm can not freely optimize their price (Calvo, 1983). If a firm can not readjust their price it is indexed to a weighted average of steady state inflation and last period's inflation. For firms that are able to choose the optimal price, $P_t^*(i)$, solve the following maximization problem:

$$\max_{P_t^*(i)} \quad E_t \sum_{s=1}^{\infty} (\xi_p \beta)^s \Lambda_{t+s}[(P_{t+s}(i) - MC_{t+s})Y_{t+s}(i)] + \Lambda_t[(P_t(i) - MC_t)Y_t(i)] \tag{1.12}$$

$s.t$ equation 1.9 and

$$P_{t+s}(i) = \prod_{k=1}^{s} \left(\pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p}\right) P_t(i) \tag{1.13}$$

where MC_t is the firms' marginal cost and equation (1.13) is the price indexation rule. Since all firms have the identical maximization problem firm indexation may be dropped from this time forth.

1.1.5 Capital Producers

Capital goods are produced in a perfectly competitive sector of the economy by purchasing final good output and transforming it into new capital. In addition to producing new capital, capital producers also buy and sell capital from entrepreneurs at price Q_t. At the end of time t capital producers purchase non-depreciated $t-1$ physical capital from entrepreneurs and investment goods from the final good producers and convert them to the time t capital stock. The time t physical capital stock is then purchased by entrepreneurs and used in time $t+1$ production. The physical capital stock evolves according to:

$$\bar{K}_t = (1-\tau)\bar{K}_{t-1} + \mu_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \tag{1.14}$$

where τ is the depreciation rate and I_t is the investment good purchased.

Capital producers face a stochastic exogenous process μ_t that alters the ability of producers to turn investment purchases into physical capital. In addition, capital producers face investment adjustment costs represented by the function S. Where $S(1) = S'(1) = 0$, $S'() > 0$ and $S''() > 0$.

1.1.6 Entrepreneurs and Banks

There exists a continuum of finite lived entrepreneurs indexed by e who are able to borrow from the perfectly competitive banking sector who obtain deposits from the households. At the end of period $t-1$, entrepreneurs buy physical capital $Q_{t-1}\bar{K}_{t-1}$ using their own nominal net worth N_{t-1} and a loan from the banking sector B^b_{t-1}.

$$Q_{t-1}\bar{K}_{t-1}(e) = B^b_{t-1}(e) + N_{t-1}(e) \tag{1.15}$$

In period t the entrepreneur is then subject to a stochastic 'productivity' shock $w_t(e)$ that increases or decreases the entrepreneur's physical capital stock. The productivity shock is drawn from the lognormal cumulative distribution $F(w)$ with mean $m_{w,t-1}$ and variance $\sigma^2_{w,t-1}$. The distribution is assumed to be known at $t-1$ and $m_{w,t-1}$ is such that $E[w_t(e)] = 1$. The standard deviation σ_w will follow an exogenous process and be considered as a financing shock as it will either increase or decrease the riskiness of loans. Entrepreneurs then choose the optimal utilization rate u_t that maximizes their time t profit.

$$\max_{u_t(e)} \quad \left[R^k_t u_t(e) - P_t a(u_t(e))\right] w_t(e)\bar{K}_{t-1}(e) \tag{1.16}$$

where R_t^k is the rental rate of utilized capital paid by the intermediate firms and $a()$ is the cost of capital utilization payed in final good output, with $a(u) = 0$, $a'() > 0$ and $a''() > 0$.

Entrepreneurs at the end of period t sell the non-depreciated physical capital to the capital producers resulting in the following period t revenue for entrepreneur e:

$$w_t(e)\tilde{R}_t^k(e)Q_{t-1}\bar{K}_{t-1}(e) \tag{1.17}$$

where

$$\tilde{R}_t^k(e) = \frac{R_t^k u_t(e) + (1-\tau)Q_t - P_t a(u_t(e))}{Q_{t-1}} \tag{1.18}$$

Entrepreneurs and banks agree upon a loan contract that consists of the size of the loan B_t^b, the interest rate of the loan R_t^c and the default threshold of the loan $\bar{w}_t$ below which entrepreneurs cannot pay back the loan and are obligated to turn over their time t revenues to the bank. However, banks are only able to recover a $(1-\mu)$ fraction of the defaulted revenue do to unmodeled bankruptcy costs.

$$\bar{w}_t(e)\tilde{R}_t^k Q_{t-1}\bar{K}_{t-1}(e) = R_t^c(e)B_{t-1}^b(e) \tag{1.19}$$

Banks abide by a zero profit condition since they operate in a perfectly competitive environment given by:

$$\begin{aligned}[1 - F_{t-1}(\bar{w}_t(e))]R_t^c(e)B_{t-1}^b(e) + (1-\mu)\int_0^{\bar{w}_t(e)} w dF_{t-1}(w)\tilde{R}_t^k Q_{t-1}\bar{K}_{t-1}(e) \\ = R_{t-1}^d B_{t-1}^b(e)\end{aligned} \tag{1.20}$$

where the first term on the left equals the expected revenue payed back to the banks, the second term equals the expected revenue a bank receives when a entrepreneur defaults and the term right of the equality is the amount paid by the bank to depositors held by the households. The optimal contract maximizes expected entrepreneur profits subject to the banks' zero profit condition and is laid out in more detail in online appendix.

The aggregate equity, V_t, of entrepreneurs operating in the economy evolves according to

$$V_t = \tilde{R}_t^k Q_{t-1}\bar{K}_{t-1} - \left(R_{t-1} + \mu G_{t-1}(\bar{w}_t)\tilde{R}_t^k \tfrac{Q_{t-1}\bar{K}_{t-1}}{Q_{t-1}\bar{K}_{t-1}-N_{t-1}}\right)(Q_{t-1}\bar{K}_{t-1} - N_{t-1}) \tag{1.21}$$

where the first term on the right is the time t revenue of entrepreneurs minus the interest and principle payments entrepreneurs borrowed from the banking sector. Notice that the agreed upon contract interest rate of the loan will be higher than the risk less rate, R_{t-1}. This external finance premium will be a function of bankruptcy costs and exogenous entrepreneur risk. At the end of each period a fraction $1-\gamma$ of entrepreneurs exit the economy and are replaced by new entrepreneurs. Exiting entrepreneurs transfer some fraction of their net worth to households and the remaining net worth is transferred to newly born entrepreneurs symbolized as W_t^e. Therefore aggregate net worth, N_t, evolves as:

$$N_t = \gamma V_t + W_t^e \tag{1.22}$$

1.1.7 Government Agencies

The *monetary authority* follows a generalized Taylor rule to set the nominal interest rate that adjusts due to deviations of inflation and output from their steady state levels.[1]

$$\left(\frac{R_t}{R}\right) = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi}\right)^{R_{\pi_1}}\left(\frac{Y_t}{Y}\right)^{R_{y_1}}\left(\frac{\pi_{t-1}}{\pi}\right)^{R_{\pi_2}}\left(\frac{Y_{t-1}}{Y}\right)^{R_{y_2}}\right]^{1-\rho_R} e^{\varepsilon_t^R} \tag{1.23}$$

where R is the steady state nominal interest rate and ρ_R resembles the degree of interest rate smoothing set by the monetary institution. ε_t^R is a stochastic monetary policy shock that affects the nominal interest rate. The central bank supplies the corresponding money supply demanded by the household to achieve the targeted nominal interest rate R_t.

The *fiscal authority* has the following government budget constraint and where govern-

[1] The Taylor rule used is different than the Taylor rule used by Smets and Wouters who had the monetary authority react to deviations in output and inflation from their completely flexible price equilibrium

ment purchases G_t is determined by the stochastic process $G\epsilon_t^G$ formally given by:

$$P_t G_t + R_{t-1}B_{t-1} + M_{t-1} = T_t + M_t + B_t \tag{1.24}$$

1.2 Log Linear Equations

The model is linearized around the non-stochastic steady state and then solved using the Sims (2002) method. This solution is the transition equation in the state-space set-up of Section ??. Variables denoted with a hat are defined as log deviations around the steady state. $\left(\hat{Y}_t = log\left(\frac{Y_t}{Y}\right)\right)$ Variables denoted without a time script are steady state values. In all, the model is reduced to 12 equations and eight exogenous shocks all of which are listed in this subsection.

Physical capital $\bar{K}_t$ accumulates according to:

$$\hat{\bar{K}}_t = (1-\tau)\hat{\bar{K}}_{t-1} + \tau\hat{I}_t + \tau(1+\beta)S''\hat{\varepsilon}_t^I \tag{1.25}$$

where ε_t^I is an AR(1) investment shock and τ is the depreciation rate and S'' is a parameter that governs investment adjustment costs. A large S'' implies that adjusting an investment schedule is costly.

Labor Demand is given by

$$\hat{L}_t = -\hat{w}_t + (1+\tfrac{1}{\psi})\hat{r}_t^k + \hat{\bar{K}}_{t-1} \tag{1.26}$$

where r_t^k is the real rental rate of capital and ψ is a parameter that captures utilization costs of capital. A large ψ infers that capital utilization costs are high. The economy's resource constraint and production function take the form:

$$\hat{Y}_t = C_y\hat{C}_t + I_y\hat{I}_t + \frac{r^k\bar{k}_y}{\psi}\hat{r}_t^k + \mathcal{M}_t + \hat{\varepsilon}_t^G \tag{1.27}$$

$$\hat{Y}_t = \phi\hat{\varepsilon}_t^a + \phi\alpha\hat{\bar{K}}_{t-1} + \frac{\phi\alpha}{\psi}\hat{r}_t^k + \phi(1-\alpha)\hat{L}_t \tag{1.28}$$

where C_y and I_y are the steady state ratio of consumption and investment to output and

$\mathcal{M}$ is the monitoring costs faced by banks. $\mathcal{M}$ is assumed to be negligible and is left out in the estimation process. ϕ resembles a fixed cost of production and is assumed to be greater than 1.

The Linearized Taylor Equation that determines the nominal interest rate is

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1-\rho)\left[r_{\pi_1}\hat{\pi}_t + r_{y_1}\hat{Y}_t + r_{\pi_2}\hat{\pi}_{t-1} + r_{y_2}\hat{Y}_{t-1} \right] + \hat{\varepsilon}_t^r \tag{1.29}$$

The consumption and investment transition equations are

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t[\hat{C}_{t+1}] - \frac{1-h}{(1+h)\sigma_c}\left(\hat{R}_t - E_t[\hat{\pi}_{t+1}]\right) + \hat{\varepsilon}_t^b \tag{1.30}$$

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}E_t[\hat{I}_{t+1}] + \frac{1}{(1+\beta)S''}\hat{q}_t + \hat{\varepsilon}_t^I \tag{1.31}$$

where $\hat{\varepsilon}_t^I$ and $\hat{\varepsilon}_t^b$ are exogenous stochastic stationary processes that effect the short term dynamics of consumption and investment. q_t is the relative price of capital and β is the discount rate.

The entrepreneurial return on capital is characterized by

$$\hat{\tilde{R}}_t^k - \hat{\pi}_t = \frac{1-\tau}{1-\tau+r^k}\hat{q}_t + \frac{r^k}{1-\tau+r^k}\hat{r}_t^k - \hat{q}_{t-1} \tag{1.32}$$

The model yields a phillips curve equal to:

$$\hat{\pi}_t = \frac{\beta}{1+\beta\iota_p}E_t[\hat{\pi}_{t+1}] + \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\iota_p)\xi_p}\left(\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a\right) + \hat{\varepsilon}_t^p \tag{1.33}$$

where ξ_p is the degree of price stickiness, ι_p is the degree of price indexation to last period's inflation rate and $\hat{\varepsilon}_t^a$, $\hat{\varepsilon}_t^p$ are exogenous processes that affect the productivity of production and the price mark up over marginal cost respectively.

Wages in the economy evolve according to:

$$\hat{w}_t = \frac{\beta}{1+\beta}E_t[\hat{w}_{t+1}] + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t[\hat{\pi}_{t+1}] - \frac{1+\beta\iota_w}{1+\beta}\hat{\pi}_t + \frac{\iota_w}{1+\beta}\hat{\pi}_{t-1}$$
$$- \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\left(1+\nu_l\frac{1+\lambda_w}{\lambda_w}\right)\xi_w}\left(\hat{w}_t - \nu_l\hat{L}_t - \frac{\sigma_c}{1-h}(\hat{C}_t - h\hat{C}_{t-1})\right) + \hat{\varepsilon}_t^w \tag{1.34}$$

where ξ_w is the degree of wage stickiness, ι_w is the degree of wage indexation to last period's inflation rate and $\hat{\varepsilon}_t^w$, is an exogenous process that affect monopoly power households hold over labor.

The finance market is characterized by two equations, the first being the spread of the return on capital over the risk free rate:

$$\hat{S}_t \equiv E_t\left[\hat{\tilde{R}}_{t+1}^k - \hat{R}_t\right] = \chi\left(\hat{q}_t + \hat{\bar{K}}_t - \hat{n}_t\right) + \hat{\varepsilon}_t^F \tag{1.35}$$

where χ is the elasticity of the spread with respect to the capital to net worth ratio and $\hat{\varepsilon}_t^F$ is a finance shock that effects the riskiness of entrepreneurs and thus the riskiness of banks being paid back in full.

The second financial equation contains the evolutional behavior of entrepreneur net worth:

$$\hat{n}_t = \delta_{\tilde{R}^k}(\hat{\tilde{R}}_t^k - \hat{\pi}_t) - \delta_R(\hat{R}_{t-1} - \hat{\pi}_t) + \delta_{qK}(\hat{q}_{t-1} + \hat{\bar{K}}_{t-1}) + \delta_n\hat{n}_{t-1} - \delta_\sigma\hat{\varepsilon}_{t-1}^F \tag{1.36}$$

where the δ coefficients are functions of the steady state values of the loan default rate, entrepreneur survival rate, the steady state variance of the entrepreneurial risk shocks, the steady state level of revenue lost in bankruptcy, and the steady state ratio of capital to net worth. The value of χ, which will be estimated, will determine the steady state level of the variance of the exogenous risk shock, the steady state value of the percentage of revenue lost in bankruptcy and the steady state level of leverage. Therefore, the value of χ will determine the values of the δ coefficients.[2] In all, the SWFF model has eight exogenous shocks, seven

[2]For a comprehensive look at the functional forms of all the δ coefficients used in coding the model, one must look at the working appendix of Del Negro and Schorfheide available at http://economics.sas.upenn.edu/ schorf/research.htm.

of which are AR(1) processes the lone exception being the monetary policy shock which is simply white noise. All processes are assumed to be i.i.d. with mean zero and standard deviation σ_i and autocorrelation parameters ρ_i, where $i = \{a, b, G, r, I, F, p, w\}$

1.3 SW Model

The SW model is identical to the SWFF model without the entrepreneur and banking sectors. Instead households own the capital, decide the utilization rate of capital, rent it to intermediate firms and sell it to capital producers. As a result the household budget constraint includes income received by renting and selling capital. In addition, households must choose how much capital to own making their complete decision set equal to $\{C_t(j), L_t(j), M_t(j), B_t(j), \bar{K}_t(j)\}_{t=0}^{\infty}$. The new household budget constraint is now

$$
\begin{aligned}
P_{t+s}C_{t+s}(j) + B_{t+s}(j) + M_{t+s}(j) \leq R_{t+s-1}B_{t+s-1}(j) + M_{t+s-1}(j) + W_{t+s}(j)L_{t+s}(j) \\
+\Pi_{t+s}(j) - T_{t+s} + \bar{K}_{t+s}(j)\left(R^k_{t+s}u_{t+s}(j) - P_{t+s}a(u_{t+s}(j))\right) \\
+P_{t+s}q_{t+s}\left((1-\tau)\bar{K}_{t+s-1}(j) - \bar{K}_{t+s}(j)\right)
\end{aligned}
\tag{1.37}
$$

The linearized first order condition of capital is given by

$$
\hat{q}_t = -(\hat{R}_t - E_t[\hat{\pi}_{t+1}]) + \frac{1-\tau}{1-\tau+r^k}E_t[\hat{q}_{t+1}] + \frac{r^k}{1-\tau+r^k}E_t[\hat{r}^k_{t+1}] + \hat{\varepsilon}^Q_t \tag{1.38}
$$

This equation will replace the linearized equation (1.32). Since the equations (1.35) and (1.36) do not exist in the SW model there is a loss of an exogenous shock. In order to be able to directly compare misspecification error of the two models it is best that both models have the same amount of exogenous shocks. This is accomplished by adding a idiosyncratic equity premium price shock represented by $\hat{\varepsilon}^Q_t$ to replace the finance shock $\hat{\varepsilon}^F_t$ of the SWFF Model. Equation (1.38) is nested in the SWFF model if there exists no finance spread (i.e $\hat{\hat{R}}^k_{t+1} = R_t$). This implies (1.32) forwarded ahead one period is identical to (1.38).

References

Bernanke, B. S., Gertler, M. and Gilchrist, S. (1999), The Financial Accelerator in a Quantitative Business Cycle Framework, *in* J. B. Taylor and M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1 of *Handbook of Macroeconomics*, Elsevier, chapter 21, pp. 1341–1393.

Christiano, L., Rostagno, M. and Motto, R. (2010), Financial Factors in Economic Fluctuations, Working Paper Series 1192, European Central Bank.

Del Negro, M. and Schorfheide, F. (2012), DSGE Model-based Forecasting, Staff Reports 554, Federal Reserve Bank of New York.

Justiniano, A., Primiceri, G. and Tambalotti, A. (2011), 'Investment Shocks and the Relative Price of Investment', *Review of Economic Dynamics* **14**(1), 101–121.

Smets, F. and Wouters, R. (2003), 'An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area', *Journal of the European Economic Association* **1**(5), 1123–1175.

Smets, F. and Wouters, R. (2007), 'Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach', *American Economic Review* **97**(3), 586–606.

A Appendix: First Order Conditions and Optimization Problems

A.1 Households and Employment Agencies

Notice that household indexation is dropped because of the existence of state-contingent securities.

- FOC for Consumption

$$b_t(C_t - hC_{t-1})^{-\sigma_c} = P_t\lambda_t = \Lambda_t \tag{A.1}$$

- FOC for Money

$$b_t\left(\frac{M_t}{P_t}\right)^{-1} = \Lambda_t - \beta E_t[\Lambda_{t+1}\pi_{t+1}^{-1}] \tag{A.2}$$

- FOC for Bonds

$$\Lambda_t = \beta R_t E_t[\Lambda_{t+1}\pi_{t+1}^{-1}] \tag{A.3}$$

- Profit maximization problem for the Employment Agency

$$\max_{L_t(j)} \quad W_t\left(\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}}\,dj\right)^{1+\lambda_{w,t}} - \int_0^1 W_t(j)L_t(j)dj \tag{A.4}$$

- Zero Profit condition for Employment Agencies

$$W_tL_t = \int_0^1 W_t(j)L_t(j)dj \tag{A.5}$$

- FOC for Wage Maximization Problem

$$\begin{aligned} E_t\sum_{s=1}^{\infty}(\xi_w\beta)^s\Lambda_{t+s}\tilde{L}_{t+s}&\left[(1+\lambda_{w,t+s})\frac{b_{t+s}(L_{t+s})^{\nu_l}}{\Lambda_{t+s}} - \prod_{k=1}^{s}\left(\pi_{t+k-1}^{\iota_w}\pi^{1-\iota_w}\right)W_t^*\right] \\ &+ \Lambda_t\tilde{L}_t\left[(1+\lambda_{w,t})\frac{b_tL_t}{\Lambda_t} - W_t^*\right] = 0 \end{aligned} \tag{A.6}$$

- Combining equation (1.4) with the zero profit condition (A.5) gives a definition for the aggregate wage:

$$W_t = \left(\int_0^1 W_t(j)^{\frac{1}{\lambda_{w,t}}}\,dj\right)^{\lambda_{w,t}} \tag{A.7}$$

- Using equation (A.7) and dropping the household indexation, the aggregate wage index is governed by:

$$W_t = \left[(1-\xi_w)(W_t^*)^{\frac{1}{\lambda_{w,t}}} + \xi_w \left(\pi_{t-1}^{\iota_w} \pi^{1-\iota_w} W_{t-1} \right)^{\frac{1}{\lambda_{w,t}}} \right]^{\lambda_{w,t}} \tag{A.8}$$

A.2 Final Good Producers and Intermediate Good Producers

- Profit maximization problem for the Final Good Sector

$$\max_{Y_t(i)} \quad P_t \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{f,t}}} dj \right)^{1+\lambda_{f,t}} - \int_0^1 P_t(i) Y_t(i) di \tag{A.9}$$

- Zero Profit condition for the Final Good Sector

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di \tag{A.10}$$

- Combining equation (1.9) with the zero profit condition (A.10) gives a definition for the aggregate price for the composite good:

$$P_t = \left(\int_0^1 P_t(i)^{-\frac{1}{\lambda_{f,t}}} dj \right)^{-\lambda_{f,t}} \tag{A.11}$$

- Intermediate Firm Cost Minimization with respect to Labor

$$W_t = (1-\alpha)\varepsilon_t^a K_t(i)^\alpha L_t(i)^{-\alpha} \tag{A.12}$$

- Intermediate Firm Cost Minimization with respect to Capital

$$R_t^k = \alpha \varepsilon_t^a K_t(i)^{\alpha-1} L_t(i)^{1-\alpha} \tag{A.13}$$

- Using ((A.12) & (A.13)) there is a relationship between aggregate labor and capital:

$$K_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} L_t \tag{A.14}$$

- Variable Costs and Marginal Costs, where marginal cost uses (A.14)

$$\begin{aligned} VC_t &= \left(W_t + R_t^k \tfrac{K_t(i)}{L_t(i)} \right) L_t(i) \\ VC_t &= \left(W_t + R_t^k \tfrac{K_t(i)}{L_t(i)} \right) \tilde{Y}_t(i) (\varepsilon_t^a)^{-1} \left(\tfrac{K_t(i)}{L_t(i)} \right)^{-\alpha} \end{aligned} \tag{A.15}$$

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}(W_t)^{1-\alpha}(R_t^k)^\alpha(\varepsilon_t^a)^{-1} \tag{A.16}$$

- FOC for Price Optimization

$$
\begin{aligned}
&E_t \sum_{s=1}^{\infty} (\xi_p \beta)^s \Lambda_{t+s} \tilde{Y}_{t+s} \left[\prod_{k=1}^{s} \left(\pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p} \right) P_t^* - (1+\lambda_{f,t+s}) MC_{t+s} \right] \\
&\quad + \Lambda_t \tilde{Y}_t \left[P_t^* - (1+\lambda_{f,t}) MC_t \right] = 0
\end{aligned}
\tag{A.17}
$$

- The aggregate price index is governed by:

$$
P_t = \left[(1-\xi_p)(P_t^*)^{\frac{1}{\lambda_{f,t}}} + \xi_p \left(\pi_{t-1}^{\iota_p} \pi^{1-\iota_p} P_{t-1} \right)^{\frac{1}{\lambda_{f,t}}} \right]^{\lambda_{f,t}} \tag{A.18}
$$

A.3 Capital Producers

- Profit function

$$
\Pi_t^k = Q_t(\bar{K}_t - (1-\tau)\bar{K}_{t-1}) - P_t I_t \tag{A.19}
$$

- Profit maximization problem for the Capital Producers

$$
\max_{I_t} \quad E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left(\frac{Q_{t+s}}{P_{t+s}} \mu_{t+s} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} - I_{t+s} \right) \tag{A.20}
$$

- Capital Producer's FOC

$$
\begin{aligned}
\Lambda_t &= \frac{\Lambda_t Q_t \mu_t}{P_t} \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] \\
&\quad + \beta E_t \left[\frac{\Lambda_{t+1} Q_{t+1} \mu_{t+1}}{P_{t+1}} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]
\end{aligned}
\tag{A.21}
$$

A.4 Entrepreneur and Banking Sector

- FOC of Entrepreneur profit

$$
R_t^k = P_t a'(u_t(e)) \tag{A.22}
$$

- Definition of utilized capital

$$
K_t = u_t \bar{K}_{t-1} \tag{A.23}
$$

- Fraction of net capital that banks receives $\Gamma_{t-1}(\bar{w}_t)$

$$\Gamma_{t-1}(\bar{w}_t) = \bar{w}[1 - F_{t-1}(\bar{w}_t)] + G_{t-1}(\bar{w}_t) \tag{A.24}$$

$$G_{t-1}(\bar{w}_t) = \int_0^{\bar{w}_t} w dF_{t-1}(w) \tag{A.25}$$

- Expected entrepreneur profits before the realization of productivity shock

$$\int_{\bar{w}_t(e)}^{\infty} \left[w_t(e)\tilde{R}_t^k Q_{t-1}\bar{K}_{t-1}(e) - R_t^c(e)B_{t-1}^b(e) \right] dF_{t-1}(w_t(e)) \tag{A.26}$$

- Rewriting banks zero profit condition using equations (A.24) and (A.25)

$$[\Gamma_{t-1}(\bar{w}_t(e)) - \mu G_{t-1}(\bar{w}_t(e))] \frac{\tilde{R}_t^k}{R_{t-1}} Q_{t-1}\bar{K}_{t-1}(e) = Q_{t-1}\bar{K}_{t-1}(e) - N_{t-1}(e) \tag{A.27}$$

- Optimal Contract Maximization Problem

$$\begin{aligned} &\max_{\{\bar{w}_t(e), \bar{K}_{t-1}(e)\}} E_{t-1} \Bigg\{ [1 - \Gamma_{t-1}(\bar{w}_t(e))]\tilde{R}_t^k Q_{t-1}\bar{K}_{t-1}(e) \\ &+\eta_t \left[[\Gamma_{t-1}(\bar{w}_t(e)) - \mu G_{t-1}(\bar{w}_t(e))] \frac{\tilde{R}_t^k}{R_{t-1}} Q_{t-1}\bar{K}_{t-1}(e) - Q_{t-1}\bar{K}_{t-1}(e) - N_{t-1}(e) \right] \Bigg\} \end{aligned} \tag{A.28}$$

- FOC for Capital

$$E_{t-1} \left\{ [1 - \Gamma_{t-1}(\bar{w}_t(e))]\tilde{R}_t^k + \eta_t \left[[\Gamma_{t-1}(\bar{w}_t(e)) - \mu G_{t-1}(\bar{w}_t(e))] \frac{\tilde{R}_t^k}{R_{t-1}} - 1 \right] \right\} = 0 \tag{A.29}$$

- FOC for $\bar{w}_t$

$$\eta_t = \frac{\Gamma'_{t-1}(\bar{w}_t(e))}{\Gamma'_{t-1}(\bar{w}_t(e)) - \mu G'_{t-1}(\bar{w}_t(e))} R_{t-1} \tag{A.30}$$

- Combining FOC's

$$\begin{aligned} E_{t-1} \Bigg\{ &[1 - \Gamma_{t-1}(\bar{w}_t)] \frac{\tilde{R}_t^k}{R_{t-1}} + \frac{\Gamma'_{t-1}(\bar{w}_t)}{\Gamma'_{t-1}(\bar{w}_t) - \mu G'_{t-1}(\bar{w}_t)} \\ &\times \left[[\Gamma_{t-1}(\bar{w}_t) - \mu G_{t-1}(\bar{w}_t)] \frac{\tilde{R}_t^k}{R_{t-1}} - 1 \right] \Bigg\} = 0 \end{aligned} \tag{A.31}$$

and dropping indexation because equations (A.22), (A.29) & (A.30) only depend on aggregate variables

- Definition of Transfer Payments to the Household

$$Trans_t = (1-\gamma)V_t - W_t^e \tag{A.32}$$

- Credit Market Clearing Equilibrium

$$D_t = B_t = B_t^b = Q_t \bar{K}_t - N_t \tag{A.33}$$

A.5 SW Model

- FOC for Capital

$$\Lambda_t q_t = \beta E_t[\Lambda_{t+1}\left(r_{t+1}^k - a(u_{t+1}) + (1-\tau)q_{t+1}\right)] \tag{A.34}$$

A.6 Log Linearizations

$$w_t = \frac{W_t}{P_t}, \quad r_t^k = \frac{R_t^k}{P_t}, \quad m_t = \frac{M_t}{P_t}, \quad p_t^* = \frac{P_t^*}{P_t}, \quad w_t^* = \frac{W_t^*}{P_t} \quad mc_t = \frac{MC_t}{P_t} \quad q_t = \frac{Q_t}{P_t}$$

$$n_t = \frac{N_t}{P_t}, \quad v_t = \frac{V_t}{P_t}, \quad w_t^e = \frac{W_t^e}{P_t}$$

- Capital Accumulation (1.25)
  Equation (1.14) delivers the steady state relationship $I/K = \tau$ and results in

$$\hat{\bar{K}}_t = (1-\tau)\hat{\bar{K}}_{t-1} + \tau\hat{I}_t + \tau\hat{\mu}_t \tag{A.35}$$

  where using (A.73) results in equation (1.25)

- Labor Demand (1.26)
  Linearizing equations (A.14), (A.22) & (A.23) results in

$$\hat{K}_t = w_t - \hat{r}_t^k + \hat{L}_t \tag{A.36}$$

$$\hat{K}_t = \hat{u}_t + \hat{\bar{K}}_{t-1} \tag{A.37}$$

$$r^k \hat{r}_t^k = a''(u)\hat{u}_t \tag{A.38}$$

$$\implies \hat{K}_t = \frac{r^k}{a''(u)}\hat{r}_t^k + \hat{\bar{K}}_{t-1} \tag{A.39}$$

  where substitution and using (A.74) results in equation (1.26)

- Resource Contraint (1.27)
  Taking the household's budget constraint and subbing in the Government's budget

constraint yields:

$$C_t + D_t + G_t = R^d_{t-1} D_{t-1} + w_t L_t + \Pi_t + Trans_t$$

Using the definition of firms' profits $\Pi_t = Y_t - w_t L_t - r^k_t \bar{K}_{t-1}$ and equation (A.32)

$$C_t + D_t + G_t - R^d_{t-1} D_{t-1} + r^k_t u_t \bar{K}_{t-1} - ((1-\gamma)v_t - w^e_t) = Y_t$$

Substituting the credit clearing condition (A.33), the definition of net worth yields (1.22) & (1.15) yields

$$C_t + G_t + q_t \bar{K}_t - v_t - R^d_{t-1} D_{t-1} + r^k_t u_t \bar{K}_{t-1} = Y_t$$

Substituting (1.19) into the zero profit equation (1.20) and (A.26) for v_t yields

$$C_t + G_t + q_t \bar{K}_t - \tilde{R}^k_t q_{t-1} \bar{K}_{t-1} + \mathcal{M}_t + r^k_t u_t \bar{K}_{t-1} = Y_t$$

Using equation (1.18) and the fact that $q_t \bar{K}_t - q_t(1-\tau)\bar{K}_{t-1} = I_t$ yields the resource constraint:

$$C_t + G_t + I_t + a(u_t)\bar{K}_{t-1} + \mathcal{M}_t = Y_t$$

Log linearizing and using (A.75) results in equation (1.27)

- Production Function (1.28)
  Log Linearizing equation (1.10), substituting in (A.39) yields

$$\hat{Y}_t = \frac{y+f}{y}\hat{\varepsilon}^a_t + \frac{y+f}{y}\alpha\hat{\bar{K}}_{t-1} + \frac{y+f}{y}\frac{r^k}{a''(u)}\alpha\hat{r}^k_t + \frac{y+f}{y}(1-\alpha)\hat{L}_t \tag{A.40}$$

  using (A.74) & (A.76) and results in equation (1.28)

- Taylor Rule (1.29)
  Taking the log of (1.23) results in equation (1.29)

- Consumption Transition (1.30)
  Linearizing (A.1) and (A.3):

$$\hat{b}_t - \frac{\sigma_c}{1-h}\hat{C}_t + \frac{h\sigma_c}{1-h}\hat{C}_{t-1} = \hat{\Lambda}_t \tag{A.41}$$

$$\hat{\Lambda}_t = \hat{R}_t + E_t[\hat{\Lambda}_{t+1}] - E_t[\hat{\pi}_{t+1}] \tag{A.42}$$

  Taking the expectation of equation (A.41) yields:

$$E_t[\hat{\Lambda}_{t+1}] = \rho_b \hat{b}_t - \frac{\sigma_c}{1-h}E_t[\hat{C}_{t+1}] + \frac{h\sigma_c}{1-h}\hat{C}_t \tag{A.43}$$

  Subbing (A.42) and (A.43) into (A.41) and using (A.77) results in equation (1.30)

- Investment Transition (1.31)
  Equation (1.31) results from log-linearizing equation (A.21) abiding by the definition $S'(1) = 0$ and (A.73)

- Entrepreneur Return on Capital (1.32)
  Putting entrepreneurial return on capital (1.18) into real terms

$$\tilde{R}_t^k = \frac{r_t^k u_t + (1-\tau)q_t - a(u_t)}{q_{t-1}} \pi_t \tag{A.44}$$

  Equation (A.44) yields the steady state identity (where $q = 1$ and a(u)=0)

$$\tilde{R}^k = (r^k + (1-\tau))\pi \tag{A.45}$$

  Log Linearizing (A.44) and using (A.45) results in (1.32)

- New Keynesian Philips Curve (1.33)
  The Philips curve is derived from the following 3 equations:

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}(w_t)^{1-\alpha}(r_t^k)^{\alpha}(\varepsilon_t^a)^{-1} \tag{A.46}$$

$$1 = \left[(1-\xi_p)(p_t^*)^{\frac{1}{\lambda_{f,t}}} + \xi_p \left(\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\pi_t^{-1}\right)^{\frac{1}{\lambda_{f,t}}}\right]^{\lambda_{f,t}} \tag{A.47}$$

$$\begin{aligned} E_t \sum_{s=0}^{\infty} (\xi_p \beta)^s \Lambda_{t+s} \tilde{Y}_{t+s} \Bigg[\prod_{k=1}^{s} \left(\left(\frac{\pi_{t+k-1}}{\pi}\right)^{\iota_p} \left(\frac{\pi_{t+k}}{\pi}\right)^{-1} \right) p_t^* \\ - (1+\lambda_{f,t+s}) mc_{t+s} \Bigg] = 0 \end{aligned} \tag{A.48}$$

  Log-linearizing the above equations results in

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha \hat{r}_t^k - \hat{\varepsilon}_t^a \tag{A.49}$$

$$\hat{p}_t^* = \frac{\xi_p}{1-\xi_p}(\hat{\pi}_t - \iota_p \hat{\pi}_{t-1}) \tag{A.50}$$

$$E_t \sum_{s=0}^{\infty} (\xi_p \beta)^s \left[\hat{p}_t^* + \hat{\Pi}_{t,t+s} - \frac{\lambda_f}{1+\lambda_f}\hat{\lambda}_{f,t+s} - \hat{mc}_{t+s}\right] = 0 \tag{A.51}$$

$$\hat{\Pi}_{t,t+s} = \sum_{k=1}^{s} \iota_p \hat{\pi}_{t+k-1} - \hat{\pi}_{t+k} \quad \text{when } s = 0,\ \hat{\Pi}_{t,t+s} = 0 \tag{A.52}$$

Solving for $\hat{p}_t^*$ and eliminating the summation of (A.51)

$$\frac{1}{1-\xi_p\beta}\hat{p}_t^* = \frac{\lambda_f}{1+\lambda_f}\hat{\lambda}_{f,t} + \hat{m}c_{t+s} - \frac{\xi_p\beta}{1-\xi_p\beta}\hat{\Pi}_{t,t+1}$$
$$+ \xi_p\beta E_t\sum_{s=1}^{\infty}(\xi_p\beta)^{s-1}\left[-\hat{\Pi}_{t+1,t+s} + \frac{\lambda_f}{1+\lambda_f}\hat{\lambda}_{f,t+s} + \hat{m}c_{t+s}\right]$$
$$\frac{1}{1-\xi_p\beta}E_t\hat{p}_{t+1}^* = E_t\sum_{s=0}^{\infty}(\xi_p\beta)^s\left[-\hat{\Pi}_{t+1,t+1+s} + \frac{\lambda_f}{1+\lambda_f}\hat{\lambda}_{f,t+1+s} + \hat{m}c_{t+1+s}\right]$$

These equations imply

$$\frac{1}{1-\xi_p\beta}\hat{p}_t^* = \frac{\lambda_f}{1+\lambda_f}\hat{\lambda}_{f,t} + \hat{m}c_{t+s} + \frac{\xi_p\beta}{1-\xi_p\beta}E_t\left[\hat{p}_{t+1}^* - \hat{\Pi}_{t,t+1}\right]$$

Plugging in the forward expectations from equations (A.50) and (A.52)

$$\frac{1}{1-\xi_p\beta}\hat{p}_t^* = \frac{\lambda_f}{1+\lambda_f}\hat{\lambda}_{f,t} + \hat{m}c_{t+s} + \frac{(\xi_p\beta)}{(1-\xi_p\beta)(1-\xi_p)}E_t\left[\hat{\pi}_{t+1}\right] - \frac{(\xi_p\beta)}{(1-\xi_p)(1-\xi_p\beta)}\iota_p\hat{\pi}_t$$

Substituting (A.50) and (A.49) into the above equation solving for $\hat{\pi}_t$ and using (A.78) results in (1.33)

- New Keynesian Wage Philips Curve (1.34)
  The Wage Philips curve is derived from the following 4 equations:

$$b_t(C_t - hC_{t-1})^{-\sigma_c} = \Lambda_t \tag{A.53}$$

$$w_t = \left[(1-\xi_w)(w_t^*)^{\frac{1}{\lambda_{w,t}}} + \xi_w\left(\pi_{t-1}^{\iota_w}\pi^{1-\iota_w}\pi_t^{-1}w_{t-1}\right)^{\frac{1}{\lambda_{w,t}}}\right]^{\lambda_{w,t}} \tag{A.54}$$

$$\tag{A.55}$$

$$E_t\sum_{s=0}^{\infty}(\xi_w\beta)^s\Lambda_{t+s}\tilde{L}_{t+s}\left[(1+\lambda_{w,t+s})\frac{b_{t+s}(L_{t+s})^{\nu_l}}{\Lambda_{t+s}} - \prod_{k=1}^{s}\left(\left(\frac{\pi_{t+k-1}}{\pi}\right)^{\iota_w}\left(\frac{\pi_{t+k}}{\pi}\right)^{-1}\right)w_t^*\right] = 0 \tag{A.56}$$

$$\tilde{L}_{t+s} = \left(\frac{\prod_{k=1}^{s}\left(\left(\frac{\pi_{t+k-1}}{\pi}\right)^{\iota_w}\left(\frac{\pi_{t+k}}{\pi}\right)^{-1}\right)w_{t+s}^*}{w_t}\right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s} \tag{A.57}$$

Log-linearizing the above equations results in

$$\hat{b}_t - \frac{\sigma_c}{1-h}\hat{C}_t + \frac{h\sigma_c}{1-h}\hat{C}_{t-1} = \hat{\Lambda}_t \tag{A.58}$$

$$\hat{w}_t^* = \frac{\xi_w}{1-\xi_w}(\hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t - \iota_p\hat{\pi}_{t-1}) \tag{A.59}$$

$$E_t\sum_{s=0}^{\infty}(\xi_p\beta)^s\left[\hat{w}_t^* + \hat{\Pi}_{t,t+s}^w - \frac{\lambda_w}{1+\lambda_w}\hat{\lambda}_{w,t+s} - \hat{b}_{t+s} - \nu_l\hat{\hat{L}}_{t+s} + \hat{\Lambda}_{t+s}\right] = 0 \tag{A.60}$$

$$\hat{\Pi}_{t,t+s}^w = \sum_{k=1}^{s}\iota_w\hat{\pi}_{t+k-1} - \hat{\pi}_{t+k} \quad \text{when } s=0,\ \hat{\Pi}_{t,t+s}^w = 0 \tag{A.61}$$

$$\hat{\hat{L}}_{t+s} = \hat{L}_{t+s} - \left(\tfrac{1+\lambda_w}{\lambda_w}\right)(\hat{w}_t^* + \hat{\Pi}_{t+s}^w - \hat{w}_{t+s}) \tag{A.62}$$

By plugging in the definition of marginal utility (A.58) and labor demand (A.62) into the wage setting FOC (A.60) and then using this equation with equations (A.59) and (A.79) one can obtain equation (1.34)

- Spread between the return on capital and the risk free rate (1.35)
  Linearizing the combined FOC of the optimal contract (A.31) and the banks' zero profit condition (A.27)

$$E_t\left[\hat{\hat{R}}_{t+1}^k - \hat{R}_t\right] + \delta_{b,w}E_t[\hat{\bar{w}}_{t+1}] + \delta_{b,\sigma_w}\hat{\sigma}_{w,t} = 0 \tag{A.63}$$

$$\hat{\hat{R}}_t^k - \hat{R}_{t-1} + \delta_{z,w}\hat{\bar{w}}_t + \delta_{z,\sigma_w}\hat{\sigma}_{w,t-1} = \frac{N}{K-N}(\hat{q}_{t-1} + \hat{\hat{K}}_{t-1} - \hat{n}_{t-1}) \tag{A.64}$$

  Solving the latter equation for $\hat{\bar{w}}_t$ and taking the forwarded expectation and plugging it into (A.63) and using (A.80) one obtains (1.35). Here the δ coefficients are functions of the steady state variables of the finance sector.

- Evolution of Aggregate Net Worth (1.36)
  Log-linearizing the evolution of equity (1.21) and plugging it into the log-linearized version of equation (1.22) one can obtain (1.36) where once again the δ coefficients are functions of the steady state variables in the finance sector.

- SW Model-Equity price evolution (1.38)
  Log Linearizing the FOC of capital (A.34) and minding the steady state relationships (A.70), (A.71), (A.72) yields:

$$\hat{q}_t + (\hat{\Lambda}_t - E_t[\hat{\Lambda}_{t+1}]) = \frac{1-\tau}{1-\tau+r^k}E_t[\hat{q}_{t+1}] + \frac{r^k}{1-\tau+r^k}E_t[\hat{r}_{t+1}^k] \tag{A.65}$$

  Subbing in equation (A.42) into the above equation and adding the equity price shock results in (1.38)

- Important Steady State Relationships SWFF Model

$$R = \beta^{-1} \tag{A.66}$$

$$r^k = SR - (1-\tau) \tag{A.67}$$

$$\tilde{R}^k R^{-1} = S \tag{A.68}$$

$$a'(u) = r^k \tag{A.69}$$

- Important Steady State Relationships SW Model

$$R = \beta^{-1} \tag{A.70}$$

$$r^k = \beta^{-1} - (1-\tau) \tag{A.71}$$

$$a'(u) = r^k \tag{A.72}$$

- **Normalizations**

$$\hat{\varepsilon}_t^I = \frac{1}{(1+\beta)S''}\hat{\mu}_t \tag{A.73}$$

$$\psi = \left(\frac{r^k}{a''(u)}\right)^{-1} \tag{A.74}$$

$$\hat{\varepsilon}_t^G = \frac{G}{Y}\epsilon_t^G \tag{A.75}$$

$$\phi = \frac{y+f}{y} \tag{A.76}$$

$$\hat{\varepsilon}_t^b = \frac{(1-h)(1-\rho_b)}{(1+h)\sigma_c}\hat{b}_t \tag{A.77}$$

$$\hat{\varepsilon}_t^P = \frac{(1-\xi_p)(1-\xi_p\beta)\lambda_f}{\xi_p(1+\beta\iota_p)(1+\lambda_f)}\hat{\lambda}_{f,t} \tag{A.78}$$

$$\hat{\varepsilon}_t^W = \frac{(1-\xi_w)(1-\xi_w\beta)\lambda_w}{(1+\beta)\xi_p(1+\nu_l\frac{1+\lambda_w}{\lambda_w})(1+\lambda_w)}\hat{\lambda}_{w,t} \tag{A.79}$$

$$\hat{\varepsilon}_t^F = \frac{\frac{\delta_{b,w}}{\delta_{z,w}}\delta_{z,\sigma_w} - \delta_{b,\sigma_w}}{1-\frac{\delta_{b,w}}{\delta_{z,w}}}\hat{\sigma}_{w,t} \tag{A.80}$$

Figure 1: Economic Agents and Interactions

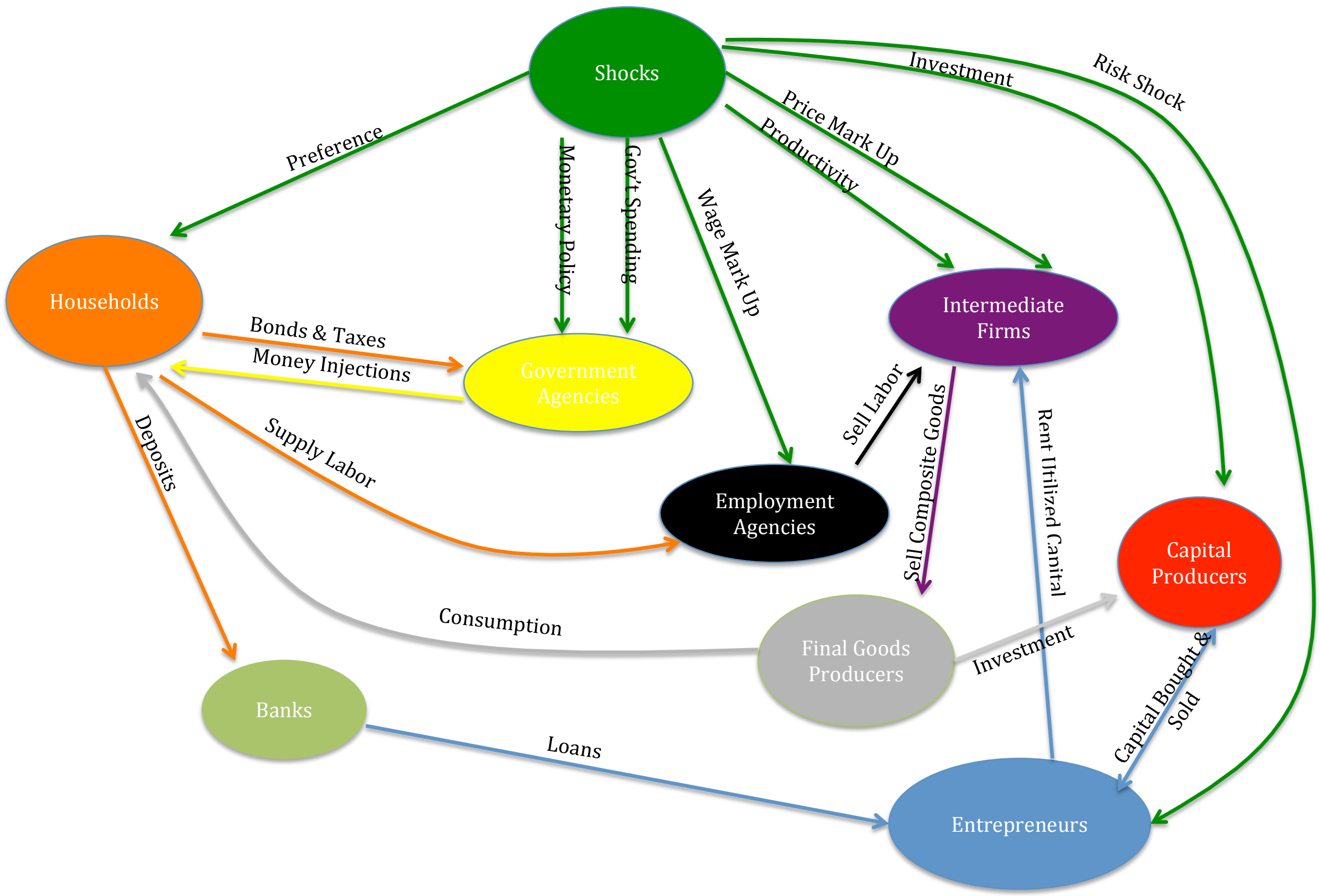

Figure 2: Sequence of Events between Agents in the Finance and Capital Sector of the Economy